

# Fixed Point Results in Fuzzy Metric Spaces, Fuzzy 2 & 3 Metric Spaces

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**Abstract:** Fixed point theory is one of the most dynamic areas of research for the last hundred years with many applications in various fields of pure and applied mathematics as well as in physical, economics and life sciences. Fixed point theory plays very important role dynamic and linear programming. Fixed point results are proved by many mathematicians in last four to five decades and further scopes are there to enhance these results. In recent years, we observed the fuzzification in almost every direction of mathematics such as arithmetic, topology, graph theory, probability theory, Image processing, stochastic processes and logic theory etc. It may not be wonder that fuzzy fixed point theory has become an area of interest for mathematicians. There are many view points of the notion of metric space in fuzzy topology. Many mathematicians used different conditions on self-mappings and proved several fixed point theorems for contractions in fuzzy metric spaces. We are interested in the results in which the distance between the objects is fuzzy, the objects themselves may be fuzzy or not. In this research article we prove some fixed point theorems for fuzzy 2-metric and fuzzy 3-metric spaces.

**Keywords:** Fixed Point Theorem, Metric Space, Ordered Set, T-norm, Fuzzy Metric Space

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## 1. Introduction

L. Zadeh [30] perhaps was the first mathematician who introduced the concept of fuzzy set in 1965 and that led to a rich growth of fuzzy mathematics [28]. Many authors namely Deng [8], Erceg, M. A. [10], Kramosil and Michalek [23] have introduced the concept of fuzzy metric space in different ways.

There are several other authors studied the fixed point theory in fuzzy metricspaces viz. [1, 6, 9, 11, 17, 19-21, 25] and for fuzzy mapping [2-5, 18, 25]. Lee *et al.* [24] proved fixed point results in Menger space in Fuzzy mapping. Recently Sabri *et al.* [26] established some properties in Fuzzy Metric Spaces.

There are many view points of the notion of metric space in fuzzy topology. We are interested in the results in which the distance between the objects is fuzzy, the objects themselves may be fuzzy or not. The most interesting

references in this direction are [8, 21, 22]. Gähler in a series of papers [13-15] investigated 2-metricspaces.

Our aim in this paper to establish results on three mappings in Fuzzy Metric spaces.

## 2. Preliminaries

Now, we are going to give some known definitions and preliminary concepts about Fuzzy Metric Space.

**Definition 2.1** [27]: A binary operation  $\circ : [0,1] \times [0,1] \rightarrow [0,1]$  is called a continuous t-norm if  $([0,1], \circ)$  is an abelian topological monoid with unit 1 such that  $a \circ b \leq c \circ d$  whenever  $a \leq c$  and  $b \leq d \quad \forall a, b, c, d \in [0,1]$ . Examples of t-norm are  $a \circ b = ab$  and  $a \circ b = \min(a, b)$ .

**Definition 2.2** [23]: The three -tuple  $(X, M, \circ)$  is called fuzzy metric space if  $X$  is an arbitrary set,  $M$  is a fuzzy set in

$X \times X \times [0, \infty)$  satisfying the following conditions:

$$Tx_{2n} = Ax_{2n-1} \quad M(x, y, 0) = 0,$$

$$(FM^1 - 2) \quad M(x, y, t) = 1 \text{ for all } t > 0 \text{ if and only if } x = y,$$

$$(FM^1 - 3) \quad M(x, y, t) = M(y, x, t),$$

$$(FM^1 - 4) \quad M(x, y, t) * M(y, z, s) \leq M(x, z, t + s),$$

$$(FM^1 - 5) \quad M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1] \text{ is left continuous } \forall \\ x, y, z \in X \text{ and } t, s > 0.$$

Then  $M$  is called a fuzzy metric on  $X$  and  $M(x, y, t)$  denotes the degree of nearness between  $x$  and  $y$  with respect to  $t$ .

Example 2.3 [16] (Induced fuzzy metric): Let  $(X, \partial)$  be a metric space, define  $aob = ab$  (or  $aob = \min\{a, b\}$ ) for all  $a, b \in [0, 1]$  and Let  $M_\partial$  be fuzzy set on  $X^2 \times [0, \infty)$  defined as.

$$M_\partial(x, y, t) = \frac{t}{t + \partial(x, y)} \quad (1)$$

$$\forall x, y \in X \text{ and } t > 0.$$

Then  $(X, M_\partial, o)$  is a fuzzy metric space. We call this fuzzy metric  $M_\partial$  induced by the metric  $\partial$  as the standard intuitionistic fuzzy metric.

$$M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t) \quad (2)$$

for all  $t > 0$  and  $n = 1, 2, \dots$ , then  $\{x_n\}$  is a Cauchy sequence in  $X$ .

Proof: For  $t > 0$  and  $q \in (0, 1)$ , we have  $M(x_2, x_3, qt) \geq M(x_1, x_2, t) \geq M(x_0, x_1, t/q)$

$$\text{or } M(x_2, x_3, qt) \geq M(x_0, x_1, t/q^2)$$

by simple induction with the condition (2) [17], we have for all  $t > 0$  and  $n = 1, 2, \dots$

$$M(x_{n+1}, x_{n+2}, t) \geq M(x_1, x_2, t/q^n) \quad (3)$$

Thus by (3) and condition  $(FM^1 - 4)$  for any positive integer  $p$  and real number  $t > 0$ , we have

$$\begin{aligned} M(x_n, x_{n+p}, t) &\geq M(x_n, x_{n+1}, t/p) o \dots p \text{ times } o' M(x_{n+p-1}, x_{n+p}, t/p) \\ &\geq M(x_1, x_2, t/pq^{n-1}) o \dots p \text{ times } o' M(x_1, x_2, t/pq^{n+p-2}) \end{aligned}$$

Therefore, by  $(FM^1 - 6)$  we have,

$$\lim_{n \rightarrow \infty} M(x_n, x_{n+p}, t) \geq lo \dots p \text{ times } o' 1 \geq 1.$$

This implies that  $\{x_n\}$  is a Cauchy sequence in  $X$ .

Lemma 2.8 [25]: If for all  $x, y \in X$ ,  $t > 0$  and for a number  $q \in (0, 1)$ ,  $M(x, y, qt) \geq M(x, y, t)$ , then  $x = y$ .

Lemmas 1, 2, 3 and Remark (1) hold for fuzzy 2 – metric spaces and fuzzy 3 – metric spaces also.

Definition 2.9: A function  $M$  is continuous in fuzzy metric space if and only if whenever  $x_n \rightarrow x$ ,  $y_n \rightarrow y$  then

Lemma 2.4 [17]: For all  $x, y \in X$ ,  $M(x, y, \cdot)$  is non-decreasing.

Definition 2.5 [17]: Let  $(X, M, o)$  be a fuzzy metric space. Then

A sequence  $\{x_n\}$  in  $X$  is said to be convergent to a point  $x \in X$ , denoted by

$$\lim_{n \rightarrow \infty} x_n = x \text{ if } t > 0 \quad \forall t > 0.$$

A sequence  $\{x_n\}$  in  $X$  is called a Cauchy sequence if

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \quad \forall t > 0 \text{ and } p > 0.$$

A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.6: Since 'o' is continuous, it follows from  $(FM^1 - 4)$ , i.e.

$M(x, y, t) o M(y, z, s) \leq M(x, z, t + s)$ , that the limit of the sequence in fuzzy metric space is uniquely determined.

Let  $(X, M, o)$  is a fuzzy metric space with the following condition.

$$(FM^1 - 6) \quad \lim_{t \rightarrow \infty} M(x, y, t) = 1 \quad \forall x, y \in X.$$

Lemma 2.7 [7]: Let  $\{x_n\}$  be a sequence in a fuzzy metric space  $(X, M, o)$  with condition  $(FM^1 - 6)$ . If there exists a number  $q \in (0, 1)$  such that

$\lim_{n \rightarrow \infty} M(x_n, y_n, t) = M(x, y, t)$ , for each  $t > 0$ .

Definition 5: Two mappings  $A$  and  $B$  on fuzzy metric space  $X$  are weakly commuting if and only if

$$M(ABu, BAu, t) \geq M(Au, Bu, t), \text{ for all } u \in X \text{ and } t > 0.$$

Definition 2.10: A binary operation  $\circ : [0,1] \times [0,1] \rightarrow [0,1]$  is called a continuous t-norm if  $([0,1], \circ)$  is an abelian topological monoid with unit 1 such that

$$a_1 \circ b_1 \circ c_1 \leq a_2 \circ b_2 \circ c_2,$$

Whenever  $a_1 \leq a_2$ ,  $b_1 \leq b_2$ ,  $\lim_{n \rightarrow \infty} M(x_n, y_n, a, b, t) = M(x, y, a, b, t)$  for  $a_1, a_2, b_1, b_2$  and  $c_1, c_2$  are in  $[0, 1]$ .

Definition 2.11: The 3- tuple  $(X, M, \circ)$  is called a fuzzy 2-metric space if  $X$  is an arbitrary set,  $\circ$  is a continuous t-norm and  $M$  is a fuzzy set in  $X \times X \times X \times [0, \infty)$  satisfying the following conditions; for all  $x, y, z, u \in X$  and  $t_1, t_2, t_3 > 0$ .

$$(FM^2 - 1) \quad M(x, y, z, 0) = 0,$$

$$(FM^2 - 2) \quad M(x, y, z, t) = 1, t > 0$$

and when at least two of the three points are equal,

$$(FM^2 - 3) \quad M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t), \text{ (Symmetry about three variables)}$$

$$(FM^2 - 4) \quad M(x, y, z, t_1 + t_2 + t_3) \geq M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3),$$

This corresponds to tetrahedron inequality in 2-metric space.

The function value  $M(x, y, z, t)$  may be interpreted as the probability the area of triangle is less than 1.

$$(FM^2 - 5) \quad M(x, y, z, \cdot) : [0,1] \rightarrow [0,1] \text{ is left continuous.}$$

Definition 2.12: Let  $(X, M, \circ)$  be a fuzzy 2-metric space, then

A sequence  $\{x_n\}$  in fuzzy 2-metric space  $X$  is said to be convergent to a point  $x \in X$  if  $\lim_{n \rightarrow \infty} M(x_n, x, a, t) = 1 \quad \forall a \in X$  and  $t > 0$ .

A sequence  $\{x_n\}$  in fuzzy 2-metric space  $X$  is called a Cauchy sequence, if  $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, a, t) = 1$ ,

$$\forall a \in X \text{ and } t > 0, p > 0.$$

A fuzzy 2-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.13: A function  $M$  is continuous in fuzzy 2-metric space if and only if whenever  $x_n \rightarrow x$ ,  $y_n \rightarrow y$  then

$$\lim_{n \rightarrow \infty} M(x_n, y_n, a, t) = M(x, y, a, t), \text{ for all } a \in X \text{ and } t > 0.$$

Definition 2.14: Two mapping  $A$  and  $B$  on fuzzy 2-metric space  $X$  are weakly commuting if and only if  $M(ABu, BAu, a, t) \geq M(Au, Bu, a, t)$  for all  $u, a \in X$  and  $t > 0$ .

Definition 2.15: A binary operation  $\circ : [0,1] \times [0,1] \times [0,1] \times [0,1] \rightarrow [0,1]$  is called a continuous t-norm if  $([0,1], \circ)$  is an abelian topological monoid with unit 1 such that

$$a_1 \circ b_1 \circ c_1 \circ d_1 \leq a_2 \circ b_2 \circ c_2 \circ d_2$$

whenever  $a_1 \leq a_2$ ,  $b_1 \leq b_2$ ,  $\lim_{n \rightarrow \infty} M(x_n, y_n, a, b, t) = M(x, y, a, b, t)$  and  $d_1 \leq d_2$  for  $a_1, a_2, b_1, b_2, c_1, c_2$  and  $d_1, d_2$  are in  $[0, 1]$ .

Definition 1.16 The 3- tuple  $(X, M, \circ)$  is called a fuzzy 3-metric space if  $X$  is an arbitrary set,  $\circ$  is a continuous t-norm and  $M$  is a fuzzy set in  $X \times X \times X \times [0, \infty)$  satisfying the following conditions, for all  $x, y, z, w, u \in X$  and  $t_1, t_2, t_3, t_4 > 0$ .

$$(FM^3 - 1) \quad M(x, y, z, w, 0) = 0,$$

$$(FM^3 - 2) \quad M(x, y, z, w, t) = 1 \text{ for all } t > 0,$$

Only when the three simplex  $(x, y, z, w)$  degenerate.

$$(FM^3 - 3) \quad M(x, y, z, t) = M(x, w, z, y, t) = M(y, z, w, x, t) = M(z, w, x, y, t) = \dots$$

$$(FM^3 - 4) \quad M(x, y, z, w, t_1 + t_2 + t_3 + t_4)$$

$$\geq M(x, y, z, u, t_1) o M(x, y, u, w, t_2) o M(x, u, z, w, t_3) o M(u, y, z, w, t_4)$$

$$(FM^3 - 5) \quad M(x, y, z, w) : [0, 1] \rightarrow [0, 1] \text{ is left continuous.}$$

Definition 2.17: Let  $(X, M, o)$  be a fuzzy 3-metric space, then-

A sequence  $\{x_n\}$  in fuzzy 3-metric space  $X$  is said to be convergent to a point  $x \in X$  if  $\lim_{n \rightarrow \infty} M(x_n, x, a, b, t) = 1$

$$\forall a, b \in X \text{ and } t > 0.$$

A sequence  $\{x_n\}$  in fuzzy 3-metric space  $X$  is called a Cauchy sequence, if  $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, a, b, t) = 1 \quad \forall a, b \in X$  and  $t > 0, p > 0$ .

A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.18: A function  $M$  is continuous in fuzzy 3-metric space if and only if whenever  $x_n \rightarrow x, y_n \rightarrow y$ , then  $\lim_{n \rightarrow \infty} M(x_n, y_n, a, b, t) = M(x, y, a, b, t) \quad \forall a, b \in X, t > 0$ .

Definition 2.19: Two mappings  $A$  and  $B$  on fuzzy 3-metric space are weakly commuting if  $M(ABu, BAu, a, b, t) \geq M(Au, Bu, a, b, t) \quad \forall u, a, b \in X$  and  $t > 0$ .

In 1979, Fisher, B. [12] proved the following theorem for three mappings in complete metric space.

Theorem A: Let  $S$  and  $T$  be continuous mappings of a complete metric space  $(X, d)$  into itself. Then  $S$  and  $T$  have a common fixed point in  $X$  if there exists a continuous mapping  $A$  of  $X$  into  $S(X) \cap T(X)$  which commute with  $S$  and  $T$  and satisfy,  $d(Ax, Ay) \leq \alpha d(Sx, Ty)$  for all  $x, y \in X$

and  $0 < \alpha < 1$ . Indeed  $S, T$  and  $A$  have a common unique fixed point.

Sharma, S. [29] proved the following extend theorem  $A$  to fuzzy metric space, fuzzy 2-metric space and fuzzy 3-metric space.

Theorem B: Let  $(X, M, o)$  be a complete fuzzy metric space with the condition  $(FM^1 - 6)$  and let  $S$  and  $T$  be continuous mappings of  $X$  in  $X$ , then  $S$  and  $T$  have a common fixed point in  $X$  if there exists a continuous mapping  $A$  of  $X$  into  $S(X) \cap T(X)$  which commute with  $S$  and  $T$  and,

$$M(Ax, Ay, qt) \geq \min\{M(Ty, Ay, t), M(Sx, Ax, t), M(Sx, Ty, T)\}$$

for all  $x, y \in X, t > 0$  and  $0 < q < 1$ , then  $S, T$  and  $A$  have a unique common fixed point.

### 3. Main Result

In this paper we generalize the result of Sharma, S. [28], Theorem B by using  $(FM^1 - 6)$  to fuzzy metric space, fuzzy 2-metric space and fuzzy 3-metric space.

Our first result is as follows:

Theorem 3.1: Let  $(X, M, o)$  be a complete fuzzy metric space with the condition  $(FM^1 - 6)$  and  $A$  and  $B$  be continuous mappings of  $X$  in  $X$ , then  $A$  and  $B$  have a common fixed point in  $X$  if there exists mapping  $T$  of  $X$  into  $A(X) \cap B(X)$  which commute with  $S$  and  $T$  and

$$\begin{aligned} M(Tx, Ty, qt) &\geq \min\{M(By, Ty, t), M(Ax, Tx, t), \\ &M(Ax, By, t), [M(Tx, Bx, t/2) o M(Bx, Ty, t/2)], \\ &[M(Tx, By, t/2) o M(By, Ty, t/2)], [M(Ax, Tx, t/2) o M(By, Tx, t/2)], \\ &[M(Ax, Ty, t/2) o M(By, Ty, t/2)]\} \end{aligned} \quad (4)$$

for all  $x, y \in X, t > 0$  and  $0 < q < 1$ . Then  $A, B$  and  $T$  have a unique common fixed point.

Proof: We define a sequence  $\{x_n\}$  such that

$$Tx_{2n} = Ax_{2n-1} \text{ and } Tx_{2n-1} = Bx_{2n}, \quad n = 1, 2, \dots$$

We shall prove that  $\{Tx_n\}$  is a Cauchy sequence.

Suppose  $x = x_{2n}$  and  $y = x_{2n+1}$  in (4), we write.

$$\begin{aligned}
M(Tx_{2n}, Tx_{2n+1}, qt) &\geq \min\{M(Bx_{2n+1}, Tx_{2n+1}, t), M(Ax_{2n}, Tx_{2n}, t), M(Ax_{2n}, Bx_{2n+1}, t), \\
&\quad [M(Tx_{2n}, Bx_{2n}, t/2) \circ M(Bx_{2n}, Tx_{2n+1}, t/2)], \\
&\quad [M(Tx_{2n}, Bx_{2n+1}, t/2) \circ M(Bx_{2n+1}, Tx_{2n+1}, t/2)], \\
&\quad [M(Ax_{2n}, Tx_{2n}, t/2) \circ M(Bx_{2n+1}, Tx_{2n}, t/2)], \\
&\quad [M(Ax_{2n}, Tx_{2n+1}, t/2) \circ M(Bx_{2n+1}, Tx_{2n+1}, t/2)]\} \\
&\geq \min\{M(Tx_{2n}, Tx_{2n+1}, t), M(Tx_{2n+1}, Tx_{2n}, t), M(Tx_{2n+1}, Tx_{2n}, t), \\
&\quad [M(Tx_{2n}, Tx_{2n-1}, t/2) \circ M(Tx_{2n-1}, Tx_{2n+1}, t/2)], \\
&\quad [M(Tx_{2n}, Tx_{2n}, t/2) \circ M(Tx_{2n}, Tx_{2n+1}, t/2)], \\
&\quad [M(Tx_{2n+1}, Tx_{2n}, t/2) \circ M(Tx_{2n}, Tx_{2n}, t/2)], \\
&\quad [M(Tx_{2n+1}, Tx_{2n+1}, t/2) \circ M(Tx_{2n}, Tx_{2n+1}, t/2)]\} \\
&\geq \min\{M(Tx_{2n}, Tx_{2n+1}, t), M(Tx_{2n}, Tx_{2n+1}, t), M(Tx_{2n}, Tx_{2n+1}, t) \\
&\quad M(Tx_{2n}, Tx_{2n+1}, t), M(Tx_{2n}, Tx_{2n+1}, t), \\
&\quad M(Tx_{2n}, Tx_{2n+1}, t), M(Tx_{2n}, Tx_{2n+1}, t)\} \\
&\geq \min\{M(Tx_{2n-1}, Tx_{2n}, t/q), M(Tx_{2n}, Tx_{2n-1}, t/q), \\
&\quad M(Tx_{2n-1}, Tx_{2n}, t/q), M(Tx_{2n-1}, Tx_{2n}, t/q), M(Tx_{2n-1}, Tx_{2n}, t/q), \\
&\quad M(Tx_{2n-1}, Tx_{2n}, t/q), M(Tx_{2n-1}, Tx_{2n}, t/q)\}
\end{aligned}$$

therefore

$$M(Tx_{2n}, Tx_{2n+1}, qt) \geq M(Tx_{2n-1}, Tx_{2n}, t/q)$$

By induction

$$M(Tx_{2k}, Tx_{2m+1}, qt) \geq M(Tx_{2m}, Tx_{2k-1}, t/q), \text{ for every } k \text{ and } m \text{ in } N.$$

Further, if  $2m+1 > 2k$ , then

$$M(Tx_{2k}, Tx_{2m+1}, qt) \geq M(Tx_{2k-1}, Tx_{2m}, t/q) \dots \geq M(Tx_0, Tx_{2m+1-2k}, t/q^{2k}) \quad (5)$$

If  $\geq M(Az, TTz, a, b, t)$ , then

$$M(Tx_{2k}, Tx_{2m+1}, qt) \geq M(Tx_{2k-1}, Tx_{2m}, t/q) \dots \geq M(Tx_{2k-(2m+1)}, Tx_0, t/q^{2m+1}) \quad (6)$$

By simple induction with (5) & (6) we have

$$M(Tx_n, Tx_{n+p}, qt) \geq M(Tx_0, Tx_p, t/q^n), \text{ for } n = 2k, p = 2m+1 \text{ or } n = 2k+1, p = 2m+1.$$

And by  $(FM^1-4)$ ,

$$M(Tx_n, Tx_{n+p}, qt) \geq M(Tx_0, Tx_1, t/2q^n) \circ M(Tx_1, Tx_p, t/2q^n) \quad (7)$$

if  $n = 2k, p = 2m$  or  $n = 2k+1, p = 2m$ .

For every positive integer  $p$  and  $n$  in  $N$ , by noting that

$$M(Tx_0, Tx_p, t/q^n) \rightarrow 1 \text{ as } n \rightarrow \infty$$

Thus  $\{Tx_n\}$  is a Cauchy sequence. Since the space  $X$  is complete, there exists

$$z = \lim_{n \rightarrow \infty} Tx_n \text{ and } z = \lim_{n \rightarrow \infty} Ax_{2n-1} = Tx_{2n}$$

It follows that  $Tz = Az = Bz$  and

$$\begin{aligned} M(Tz, T^2z, qt) &\geq \min \{ M(BTz, TTz, t), M(Tz, Bz, t), M(Tz, BTz, t), \\ &\quad \left[ M(Az, Bz, t/2) \circ M(Bz, TTz, t/2) \right], \left[ M(Tz, BTz, t/2) \circ M(BTz, TTz, t/2) \right], \\ &\quad \left[ M(Az, Tz, t/2) \circ M(TTz, Tz, t/2) \right], \\ &\quad \left[ M(Tz, TTz, t/2) \circ M(TTz, TTz, t/2) \right] \} \\ &\geq \min \{ 1, 1, M(Tz, BTz, t), M(Az, TTz, t), M(Tz, TTz, t), M(Az, TTz, t), M(Tz, TTz, t) \} \\ &\geq \min \{ M(Tz, TBz, t), M(Az, T^2z, t), M(Tz, T^2z, t), M(Az, T^2z, t), M(Tz, T^2z, t) \} \\ &\geq \min \{ M(Tz, T^2z, t), M(Tz, T^2z, t), M(Tz, T^2z, t), M(Tz, T^2z, t), M(Tz, T^2z, t) \} \\ &\geq M(Tz, T^2z, t) \\ &\quad \dots\dots\dots \\ &\quad \dots\dots\dots \\ &\geq M(Tz, T^2z, t/q^n) \end{aligned}$$

Since  $\lim_{n \rightarrow \infty} M(Tz, T^2z, t/q^n) = 1$ , so  $Tz = T^2z$

Thus  $z$  is a common fixed point of  $T, A$  and  $B$ .

For uniqueness, let  $(w \neq z)$  be another common fixed point of  $A, B$  and  $T$ . By (1), we write  $M(Tz, Tw, qt) \geq \min \{ M(Bw, Tw, t), M(Az, Tz, t), M(Az, Bw, t),$

$$\begin{aligned} &\left[ M(Tz, Bz, t/2) \circ M(Bz, Tw, t/2) \right], \\ &\left[ M(Tz, Bw, t/2) \circ M(Bw, Tw, t/2) \right], \\ &\left[ M(Az, Tz, t/2) \circ M(Bw, Tz, t/2) \right], \\ &\left[ M(Tz, Tw, t/2) \circ M(Bw, Tw, t/2) \right] \} \end{aligned}$$

which implies that

$$M(z, w, qt) \geq M(z, w, t)$$

Therefore by lemma 2.8, we write  $z = w$ .

This completes the proof of Theorem 1.

Now, we extend Theorem 1 for fuzzy 2-metric space.

Theorem 3.2: Let  $(X, M, o)$  be a complete fuzzy 2-metric space and let  $A$  and  $B$  be continuous mappings of  $X$  in  $X$ , then  $A$  and  $B$  have a common fixed point in  $X$  if there exists continuous mapping  $T$  of  $X$  into  $A(X) \cap B(X)$  which commute with  $A$  and  $B$  and

$$\begin{aligned} M(Tx, Ty, a, qt) &\geq \min \{ M(By, Ty, a, t), M(Ax, Tx, a, t), \\ &\quad M(Ax, By, a, t), [M(Tx, Bx, Ty, t/3) \circ M(Tx, Bx, a, t/3) \circ M(Bx, Ty, a, t/3)], \end{aligned}$$

$$\begin{aligned}
& [M(Tx, By, Ty, t/3) o (By, Ty, a, t/3) o M(Tx, By, a, t/3)], \\
& [M(Ax, Tx, By, t/3) o M(Ax, Tx, a, t/3) o M(By, Tx, a, t/3)], \\
& [M(Ax, Ty, By, t/3) o M(Ax, Ty, a, t/3) o M(By, Ty, a, t/3)] \}
\end{aligned} \tag{8}$$

For all  $x, y, a$  in  $X$ ,  $t > 0$  and  $0 < q < 1$ ,

$$\lim_{t \rightarrow \infty} M(x, y, z, t) = 1 \text{ for all } x, y, z \text{ in } X. \tag{9}$$

Then  $A$ ,  $B$  and  $T$  have a unique fixed point.

Proof: We define a sequence  $\{x_n\}$  such that

$$Tx_{2n} = Ax_{2n-1} \text{ and } Tx_{2n-1} = Bx_{2n}, n = 1, 2, \dots$$

We shall prove that  $\{Tx_n\}$  is a Cauchy sequence.

Suppose  $x = x_{2n}$ ,  $y = x_{2n+1}$ , in (8), we write

$$\begin{aligned}
M(Tx_{2n}, Tx_{2n+1}, a, qt) & \geq \min \{ M(Bx_{2n+1}, Tx_{2n+1}, a, t), M(Ax_{2n}, Tx_{2n}, a, t), M(Ax_{2n}, Bx_{2n+1}, a, t), \\
& [M(Tx_{2n}, Tx_{2n+1}, Bx_{2n}, t/3) o M(Tx_{2n}, Bx_{2n}, a, t/3) o \\
& M(Bx_{2n}, Tx_{2n+1}, a, t/3)], \\
& [M(Tx_{2n}, Bx_{2n+1}, Tx_{2n+1}, t/3) o (Bx_{2n+1}, Tx_{2n+1}, a, t/3) o M(Tx_{2n}, Bx_{2n+1}, a, t/3)], \\
& [M(Ax_{2n}, Tx_{2n}, Bx_{2n+1}, t/3) o M(Ax_{2n}, Tx_{2n}, a, t/3) o M(Bx_{2n+1}, Tx_{2n}, a, t/3)], \\
& [M(Ax_{2n}, Tx_{2n+1}, Bx_{2n+1}, t/3) o M(Ax_{2n}, Tx_{2n+1}, a, t/3) o M(Bx_{2n+1}, Tx_{2n+1}, a, t/3)] \} \\
& \geq \min \{ M(Tx_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n+1}, Tx_{2n}, a, t), M(Tx_{2n+1}, Tx_{2n}, a, t), \\
& M\left(Tx_{2n}, Tx_{2n+1}, a, \frac{t}{3} + \frac{t}{3} + \frac{t}{3}\right), M\left(Tx_{2n}, Tx_{2n+1}, a, \frac{t}{3} + \frac{t}{3} + \frac{t}{3}\right), \\
& M\left(Tx_{2n}, Tx_{2n+1}, a, \frac{t}{3} + \frac{t}{3} + \frac{t}{3}\right), M\left(Tx_{2n}, Tx_{2n+1}, a, \frac{t}{3} + \frac{t}{3} + \frac{t}{3}\right) \} \\
& \geq \min \left\{ \begin{aligned} & M(Tx_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n}, Tx_{2n+1}, a, t), \\ & M(Tx_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n}, Tx_{2n+1}, a, t), \\ & M(Tx_{2n}, Tx_{2n+1}, a, t), M(Tx_{2n}, Tx_{2n+1}, a, t) \end{aligned} \right\} \\
& \geq \min \left\{ \begin{aligned} & M(Tx_{2n-1}, Tx_{2n}, a, t/q), M(Tx_{2n-1}, Tx_{2n}, a, t/q), \\ & M(Tx_{2n-1}, Tx_{2n}, a, t/q), M(Tx_{2n-1}, Tx_{2n}, a, t/q), M(Tx_{2n-1}, Tx_{2n}, a, t/q), \\ & M(Tx_{2n-1}, Tx_{2n}, a, t/q), \\ & M(Tx_{2n-1}, Tx_{2n}, a, t/q) \end{aligned} \right\}
\end{aligned}$$

This gives

$$M(Tx_{2n}, Tx_{2n+1}, a, qt) \geq M(Tx_{2n-1}, Tx_{2n}, a, t/q)$$

By induction

$$M(Tx_{2n}, Tx_{2m+1}, a, qt) \geq M(Tx_{2m}, Tx_{2k-1}, a, t/q)$$

For every  $k$  and  $m$  in  $N$ . Further, if  $2m+1 > 2k$ , then

$$M(Tx_{2k}, Tx_{2m+1}, a, qt) \geq M(Tx_{2k-1}, Tx_{2m}, a, t/q) \dots \geq M(Tx_0, Tx_{2m+1-2k}, a, t/q^{2k}) \tag{10}$$

If  $2k > 2m + 1$ , then

$$M(Tx_{2k}, Tx_{2m+1}, a, qt) \geq M(Tx_{2k-(2m+1)}, Tx_0, a, t/q^{2m+1}) \quad (11)$$

By simple induction with (10) and (11) we have

$$M(Tx_n, Tx_{n+p}, a, qt) \geq M(Tx_0, Tx_p, a, t/q^n)$$

for  $n = 2k$ ,  $p = 2m + 1$  or  $n = 2k + 1$ ,  $p = 2m + 1$

And by  $(FM^1 - 4)$

$$M(Tx_n, Tx_{n+p}, a, qt) \geq M\left(Tx_0, Tx_p, Tx_1, \frac{t}{3q^n}\right) o M\left(Tx_0, Tx_1, a, \frac{t}{3q^n}\right) o M\left(Tx_1, Tx_p, a, \frac{t}{3q^n}\right) \quad (12)$$

if  $n = 2k$ ,  $p = 2m$  or  $n = 2k + 1$ ,  $p = 2m$ .

For every positive integer  $p$  and  $n$  in  $N$ , by noting that

$$M\left(Tx_0, Tx_p, a, \frac{t}{q^n}\right) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

Thus  $\{Tx_n\}$  is a Cauchy sequence. Since the space  $X$  is complete, there exists

$$z = \lim_{n \rightarrow \infty} Tx_n \text{ and } z = \lim_{n \rightarrow \infty} Tx_{2n-1} = Tx_{2n}$$

It follows that  $Tz = Az = Bz$  and

$$M(Tz, T^2z, a, qt) \geq \min\{M(BTz, TTz, a, t), M(Az, Tz, a, t), M(Az, BTz, a, t),$$

$$\left[ M\left(Tz, TTz, Bz, \frac{t}{3}\right) o M\left(Tz, Bz, a, \frac{t}{3}\right) o M\left(Bz, TTz, a, \frac{t}{3}\right) \right],$$

$$\left[ M\left(Tz, TTz, BTz, \frac{t}{3}\right) o M\left(Tz, BTz, a, \frac{t}{3}\right) o M\left(BTz, TTz, a, \frac{t}{3}\right) \right],$$

$$\left[ M\left(Az, Tz, BTz, \frac{t}{3}\right) o M\left(Az, Tz, a, \frac{t}{3}\right) o M\left(BTz, Tz, a, \frac{t}{3}\right) \right]$$

$$\left[ M\left(Az, TTz, BTz, \frac{t}{3}\right) o M\left(Az, TTz, a, \frac{t}{3}\right) o M\left(BTz, TTz, a, \frac{t}{3}\right) \right]$$

$$\geq M(Az, TBz, a, t)$$

$$\geq M(Tz, TTz, a, t)$$

$$\geq M(Tz, T^2z, a, t)$$

.....

.....

$$\geq M\left(Tz, T^2z, a, \frac{t}{q^n}\right).$$



Since  $\lim_{n \rightarrow \infty} M(Tz, T^2z, a, t/q^n) = 1$ , so  $Tz = T^2z$ .

Thus  $z$  is a common fixed point of  $T$ ,  $A$  and  $B$ .

For uniqueness, let  $w (w \neq z)$  be another common fixed point of  $A$ ,  $B$  and  $T$ .

By (8), we can write

$$M(Tz, Tw, a, qt) \geq \min\{M(Bw, Tw, a, t), M(Az, Tz, a, t), M(Az, Bw, a, t), \\ M(Tz, Tw, a, t), M(Tz, Tw, a, t), M(Az, Tz, a, t), M(Az, Tw, a, t)\}$$

which implies that

$$M(z, w, a, qt) \geq M(z, w, a, t)$$

Therefore by lemma 3 we write  $z = w$ .

This completes the proof of Theorem 2.

We prove the following theorem as a extension of Theorem 1 in fuzzy 3- metric space.

Theorem 3.3: Let  $(X, M, o)$  be a complete fuzzy 3 metric space and let  $A$  and  $B$  be continuous mappings of  $X$  in  $X$ , then  $A$  and  $B$  have a common fixed point in  $X$  if there exists continuous mapping  $T$  of  $X$  into  $A(X) \cap B(X)$  which commute with  $A$  and  $B$  and

$$M(Tx, Ty, a, b, qt) \geq \min\{M(By, Ty, a, b, t), M(Ax, Tx, a, b, t), M(Ax, By, a, b, t), \\ \left[ M\left(Tx, Ty, a, Bx, \frac{t}{4}\right) o M\left(Tx, Ty, Bx, b, \frac{t}{4}\right) o M\left(Tx, Bx, a, b, \frac{t}{4}\right) o \right. \\ \left. M\left(Bx, Ty, a, b, \frac{t}{4}\right) \right], \left[ M\left(Tx, Ty, a, By, \frac{t}{4}\right) o M\left(Tx, Ty, By, b, \frac{t}{4}\right) o \right. \\ \left. M\left(Tx, By, a, b, \frac{t}{4}\right) o M\left(By, Ty, a, b, \frac{t}{4}\right) \right], [M(Ax, Tx, a, By, t/4) o M(Ax, Tx, By, b, t/4) o \\ M(Ax, Tx, a, b, t/4) o M(By, Tx, a, b, t/4)], [M(Ax, Ty, a, By, t/4) o M(Ax, Ty, a, By, t/4) o \\ M(Ax, Ty, a, b, t/4) o M(By, Ty, a, b, t/4)]\} \quad (13)$$

For all  $x, y, a, b$  in  $X$ ,  $t > 0$  and  $0 < q < 1$ ,

$$\lim_{t \rightarrow \infty} M(x, y, z, w, t) = 1 \quad (14)$$

For all  $x, y, z, w$  in  $X$ .

Then  $A$ ,  $B$  and  $T$  have a unique common fixed point.

Proof: We define a sequence  $\{x_n\}$  such that  $Tx_{2n} = Ax_{2n-1}$  and  $Tx_{2n-1} = Bx_{2n}$ ,  $n = 1, 2, \dots$ .

First, we shall prove that  $\{Tx_n\}$  is Cauchy sequence.

Suppose  $x = x_{2n}$ ,  $y = x_{2n+1}$  in (13), we write

$$M(Tx_{2n}, Tx_{2n+1}, a, b, qt) \geq \min\{M(Bx_{2n+1}, Tx_{2n+1}, a, b, t), M(Ax_{2n}, Tx_{2n}, a, b, t), M(Ax_{2n}, Bx_{2n+1}, a, b, t), \\ \left[ M\left(Tx_{2n}, Tx_{2n+1}, a, Bx_{2n}, \frac{t}{4}\right) o M\left(Tx_{2n}, Tx_{2n+1}, Bx_{2n}, b, \frac{t}{4}\right) o \right. \\ \left. M\left(Tx_{2n}, Bx_{2n}, a, b, \frac{t}{4}\right) o M\left(Bx_{2n}, Tx_{2n+1}, a, b, \frac{t}{4}\right) \right],$$

$$\begin{aligned}
& \left[ M \left( Tx_{2n}, Tx_{2n+1}, a, Bx_{2n+1}, \frac{t}{4} \right) o M \left( Tx_{2n}, Tx_{2n+1}, Bx_{2n+1}, b, \frac{t}{4} \right) o \right. \\
& M \left( Tx_{2n}, Bx_{2n+1}, a, b, \frac{t}{4} \right) o M \left( Bx_{2n+1}, Tx_{2n+1}, a, b, \frac{t}{4} \right) \left. \right], [M(Ax_{2n}, Tx_{2n}, a, Bx_{2n+1}, t/4) o \\
& M(Ax_{2n}, Tx_{2n}, Bx_{2n+1}, b, t/4) o \\
& M(Ax_{2n}, Tx_{2n}, a, b, t/4) o M(Bx_{2n+1}, Tx_{2n}, a, b, t/4)], [M(Ax_{2n}, Tx_{2n+1}, a, Bx_{2n+1}, t/4) o \\
& M(Ax_{2n}, Tx_{2n+1}, Bx_{2n+1}, b, t/4) o \\
& M(Ax_{2n}, Tx_{2n+1}, a, b, t/4) o M(Bx_{2n+1}, Tx_{2n+1}, a, b, t/4)] \} \\
& \geq \min \{ M(Tx_{2n}, Tx_{2n+1}, a, b, t), M(Tx_{2n+1}, Tx_{2n}, a, b, t), \\
& M(Tx_{2n+1}, Tx_{2n}, a, b, t), M(Tx_{2n}, Tx_{2n+1}, a, b, t), \left. \begin{array}{l} M(Tx_{2n}, Tx_{2n+1}, a, b, t), \\ M(Tx_{2n}, Tx_{2n+1}, a, b, t) \end{array} \right\}
\end{aligned}$$

Which gives

$$M(Tx_{2n}, Tx_{2n+1}, a, b, qt) \geq M(Tx_{2n-1}, Tx_{2n}, a, b, t/q).$$

By induction

$$M(Tx_{2k}, Tx_{2m+1}, a, b, qt) \geq M(Tx_{2m}, Tx_{2k-1}, a, b, t/q)$$

for every  $k$  and  $m$  in  $N$ .

Further, if  $2m+1 > 2k$ , then

$$\begin{aligned}
& M(Tx_{2k}, Tx_{2m+1}, a, b, qt) \geq M(Tx_{2k-1}, Tx_{2m}, a, b, t/q) \\
& \dots\dots\dots \\
& \dots\dots\dots \\
& \geq M \left( Tx_0, Tx_{2m+1-2k}, a, b, \frac{t}{q^{2k}} \right) \tag{15}
\end{aligned}$$

If  $2k > 2m+1$ , then

$$M(Tx_{2k}, Tx_{2m+1}, a, b, qt) \geq M \left( Tx_{2k-(2m+1)}, Tx_0, a, b, \frac{t}{q^{2m+1}} \right) \tag{16}$$

By simple induction with (15) and (16) we have

$$M(Tx_n, Tx_{n+p}, a, b, qt) \geq M \left( Tx_0, Tx_p, a, b, \frac{t}{q^n} \right)$$

for  $n = 2k$ ,  $p = 2m+1$  or  $n = 2k+1$ ,  $p = 2m+1$ .

And by  $(FM^1 - 4)$

$$M(Tx_n, Tx_{n+p}, a, b, qt) \geq M \left( Tx_0, Tx_p, a, Tx_1, \frac{t}{4q^n} \right) o M \left( Tx_0, Tx_p, Tx_1, b, \frac{t}{4q^n} \right) o$$

$$M\left(Tx_0, Tx_1, a, b, \frac{t}{4q^n}\right) \circ M\left(Tx_1, Tx_p, a, b, \frac{t}{4q^n}\right) \quad (17)$$

if  $n = 2k$ ,  $p = 2m$  or  $n = 2k + 1$ ,  $p = 2m$ .

For every positive integer  $p$  and  $n$  in  $N$ , by noting that

$$M\left(Tx_0, Tx_p, a, b, \frac{t}{q^n}\right) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

Thus  $\{Tx_n\}$  is a Cauchy sequence. Since the space  $X$  is complete, there exists

$$z = \lim_{n \rightarrow \infty} Tx_n \text{ and } z = \lim_{n \rightarrow \infty} Tx_{2n-1} = Tx_{2n}.$$

It follows that  $Tz = Az = Bz$  and

$$\begin{aligned} M(Tz, T^2z, a, qt) &\geq \min\{M(BTz, TTz, a, b, t), M(Az, Tz, a, b, t), M(Az, BTz, a, b, t), \\ &\quad [M(Tz, TTz, a, b, t)]\}, \\ M(Tz, TTz, a, b, qt) &\geq \min\{M(BTz, TTz, a, b, t), M(Az, Tz, a, b, t), M(Az, BTz, a, b, t), \\ &\quad M(Tz, TTz, a, b, t), M(Tz, TTz, a, b, t), M(Az, TTz, a, b, t), \\ &\quad M(Az, TTz, a, b, t)\} \\ &\geq M(Az, TTz, a, b, t) \\ &\geq M(Tz, TTz, a, b, t/q^n) \end{aligned}$$

Since  $\lim_{n \rightarrow \infty} M(Tz, T^2z, a, b, t/q^n) = 1$

So  $Tz = T^2z$ .

Thus  $z$  is a common fixed point of  $T$ ,  $A$  and  $B$ .

For uniqueness, let  $w (w \neq z)$  be another fixed point of  $A$ ,  $B$  and  $T$ . By (13) we write

$$\begin{aligned} M(Tz, Tw, a, b, qt) &\geq \min\{M(Bw, Tw, a, b, t), M(Az, Tz, a, b, t), M(Az, Bw, a, b, t), \\ &\quad M(Tz, Tw, a, b, t), M(Tz, Tw, a, b, t), \\ &\quad M(Tz, Tw, a, b, t), M(Tz, Tw, a, b, t)\} \end{aligned}$$

which implies that

$$M(z, w, a, b, qt) \geq M(z, w, a, b, t).$$

Therefore by lemma 3, we have  $z = w$ .

This completes the proof.

## 4. Conclusion

In this work, we prove, by using Continuous mappings of Complete Metric Space to generalized Fuzzy 2-Metric Spaces and 3-Metric Spaces. From the results we proved above, we can conclude that fixed point theorems will easily be prove in Fuzzy Metric space with using fuzzy topology.

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