



# Global Exponential Stability of Periodic Solutions for Static Recurrent Neural Networks with Impulsive Finite

Xuan Guo<sup>1</sup>, Hong Zhang<sup>2, 3, 4, 5, 6, 7, 8, 9, 10</sup>, Shanzai Lee<sup>2, 3, 4, 5, 6, 7, 8, 9, 10</sup>

<sup>1</sup>School of Business Administration, Southwestern University of Finance and Economics, Chengdu, China

<sup>2</sup>School of Information, Beijing Wuzi University, Beijing, China

<sup>3</sup>Chinese Academy of Sciences, Bioinformatics Research Center, Beijing, China

<sup>4</sup>Chinese Academy of Sciences, Power System Research Center, Beijing, China

<sup>5</sup>Chinese Academy of Sciences, Partial Differential Equation and Its Application Center, Beijing, China

<sup>6</sup>Chinese Academy of Sciences, Statistical Science Research Center, Beijing, China

<sup>7</sup>Chinese Academy of Sciences, Center for Optimization and Applied Research, Beijing, China

<sup>8</sup>Chinese Academy of Sciences, Stochastic Analysis and Research Center, Beijing, China

<sup>9</sup>Chinese Academy of Sciences, Academy of Mathematics and Systems Science, Beijing, China

<sup>10</sup>School of Mathematical Sciences, Peking University, Beijing, China

## Email address:

dr.yuwenjunxian@gmail.com (Shanzai Lee)

## To cite this article:

Xuan Guo, Hong Zhang, Shanzai Lee. Global Exponential Stability of Periodic Solutions for Static Recurrent Neural Networks with Impulsive Finite. *Machine Learning Research*. Vol. 2, No. 4, 2017, pp. 119-124. doi: 10.11648/j.mlr.20170204.12

**Received:** February 7, 2017; **Accepted:** May 22, 2017; **Published:** July 17, 2017

---

**Abstract:** In this paper, we consider the sufficient conditions for the stability of periodic solutions of static recurrent neural networks with impulsive delay. In this paper, we study the time - delay static recurrent neural network affected by pulse. The results show that the neural network is stable when the pulse function is linear and relatively small, and a condition for the periodic solution with exponential stability is obtained. This paper introduces the research status of artificial neural network, summarizes the research background and development of static recurrent neural network dynamic system, and introduces the main work of this paper. Using the fixed point theory, M. The existence of periodic solutions and the global robust exponential stability of the static recursive neural network with variable delays and the existence of almost periodic solutions of the static recursive neural network of the partitioned time are studied by combining the properties of the matrix and the Lyapunov function combined with the inequality technique. Global exponential stability, the stability conditions of the corresponding problem are obtained respectively, and the results of the related research are generalized. Using Lyapunov. The stability of the quasi - static neural recursive neural network and the stability of the periodic solution are studied. The condition of the stationary static recursive neural network is obtained and the correctness of the condition is illustrated. Considering the influence of stochastic perturbation on the dynamic behavior of static recurrent neural network, the static recursive neural network with time delay and the static recursive neural network with distributed time delay are studied by using the infinitesimal operator, Ito formula and the convergence theorem of martingales. Global critical exponential stability of quasi - static neural network with stochastic perturbation. The static recursive neural network with Markovian modulation and the time-delay static recurrent neural network model considering both random perturbation and Markovian switching are studied. The linear matrix inequality, the finite state space Markov chain property and the Lyapunov-krasovskii function, The judgment condition of the global exponential stability of the system is obtained. Firstly, the global exponential stability problem of quasi - static neural neural network with time - delay and recursive neural network is studied by using the generalized Halanay inequality. Then the stability of the Markovian response sporadic static recurrent neural network is studied by combining the properties of Markov chain.

**Keywords:** Global Exponential Stability, Static Recurrent Neural Networks, Periodic Solutions, Impulsive Finite

---

### 1. Introduction

In the past two decades, the theory and application of artificial neural network have attracted the great interest of scientists and become one of the hotspots in the field of nonlinear science. This is mainly because the artificial neural network is a nonlinear information processing system, Has a wide range of applications. Neural network can be divided into engineering neural network and mathematical neural network. Engineering neural network is a kind of information processing function in hardware or software form. The mathematical neural network is a mathematical model proposed by engineering neural network, which is usually called recursive neural network, mainly to study its dynamic characteristics and provide theoretical support for engineering neural network and guarantee. According to the basic variables of the system, the mathematical model of recurrent neural network can be divided into static neural network and local neural network. At present, most researches on recurrent neural networks focus on the local neural network model, and the static model is relatively few. However, many important neural networks are attributed to the static model, therefore, the study of static model has important theoretical significance and practical value.

In the electronic implementation of artificial neural

$$\begin{cases} \dot{x}(t) = -a_i(t)x_i(t) + f_i(\sum_{j=1}^n w_{ij}(t)x_j(t) + I_i(t)) + g_i(\sum_{j=1}^n m_{ij}(t) \int_{-\tau}^0 \eta_j(\theta)x_j(t+\theta)d\theta + J_j(t)), \\ 0 \leq t \neq t_k, \\ \Delta x_i(t) = x_i(t_k+0) - x_i(t_k) = P_{ik}(x_i(t_k)), \\ k \in \{1, \dots, p\}, i = \{1, \dots, n\}. \end{cases} \tag{1}$$

in which  $a_i(t) > 0, \tau > 0$ ,  $P_{ik}$  is constant,  $\eta(\bullet)$  is Nonnegative integrable function, and notes  $\int_{-\tau}^0 \eta_j(\theta)d\theta = \Delta \eta_j > 0$ .

In whole paper, we assume that all of  $a_i(t), I_i(t), J_i(t), w_{ij}(t), m_{ij}(t)$ , are continuous function with a same period  $\omega(\omega > 0)$ .

And the following conditions hold.

$(H_1)$ : for  $i \in \{1, 2, \dots, n\}$ ,  $f_i(x), g_i(x)$  are lipschitz continuous, ie there exist constant  $\alpha_i > 0, \beta_i > 0$ , making

$$|f_i(X) - f_i(y)| \leq \alpha_i |x - y|,$$

$$|g_i(X) - g_i(y)| \leq \beta_i |x - y|,$$

$$\forall x, y \in R, i = 1, 2, \dots, n;$$

$$(H_2): -2 \leq P_{ik} \leq 0, \text{ for } i \in \{1, 2, \dots, n\}, k \in \{1, 2, \dots, p\}$$

$$(H_3): 0 < t_0 < t_1 < t_p \leq \omega \text{ (pulses having } \omega \text{ Periodicity)}$$

networks, time delay is unavoidable due to the limited conversion speed of network neuron amplifiers. Similarly, pulsed phenomena and random perturbations are inevitable in the implementation of neural networks and are widespread, and in many cases they tend to occur in the same system. Therefore, this paper studies the time-delay static recurrent neural network, Considering the influence of pulse and random disturbance on neural network.

From the point of view of biological neural network system, the human brain is often in periodic or chaotic state, so it is very important to research the periodic oscillation and chaos phenomenon of neural network, and the periodicity includes periodicity. Problem, the study of almost periodic movement is often more realistic than the research cycle of movement. On the other hand, in practical applications, the neural network system is sometimes affected by the limitations of the amplifier conversion speed, etc., often lead to drastic changes in time lag with time.

### 2. Preparations

Consider the following pulse-distributed time-delay static recurrent neural network model:

For convenience, the following notation is introduced:

$$w_{ij} = \sup_{t \in R} (|w_{ji}(t)|), \quad m_{ij} = \sup_{t \in R} (|m_{ji}(t)|)$$

$$A = \text{diag}(a_1, a_2, \dots, a_n), \quad M = (m_{ij}^+)_{n \times n}$$

$$\alpha = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_n), \quad K = \text{diag}(\eta_1, \eta_2, \dots, \eta_n)$$

$$W = (w_{ij}^+)_{n \times n}, \quad \beta = \text{diag}(\beta_1, \beta_2, \dots, \beta_n),$$

*Definition1*: function  $\phi: [-\tau, 0] \rightarrow R^n$  is called a  $C^*$ -function, if the following conditions are satisfied:

(i)  $\phi$  is a piecewise continuous function with discontinuity point of the first kind in  $t_k - \tau \in [-\tau, 0], k \in \{1, 2, \dots, p\}$ , and is continuity on the left of every discontinuity point;

(ii) for any  $i \in \{1, 2, \dots, n\}$  and  $k \in \{1, 2, \dots, p\}$ , when  $t_k - \tau \in [-\tau, 0]$ , the following equation holds:  $\phi_i(t_k - \tau + 0) = \phi_i(t_k - \tau) + P_{ik}\phi_i(t_k - \tau)$ .

Let  $C^*$  denote all  $C^*$ -function and endowed with  $\infty$ -norm, that is, for any  $\phi \in C^*$

$$\|\phi\|_\infty = \sup_{-\tau \leq \theta \leq 0} \|\phi(\theta)\|_\infty = \sup_{-\tau \leq \theta \leq 0} \max_{1 \leq i \leq n} |\phi_i(\theta)|$$

Thus  $C^*$  is a complete normed linear space- the Banach Space.

*Definition2:* a function  $x : [-\tau, \infty] \rightarrow R^n$  is called as a  $\omega$ -periodic solution for the system (3.2.1) with the initial value  $\phi \in C^*$ , if it satisfies following two conditions:

- (i) when  $t > 0$ ,  $x$  satisfies the system (3.2.1) and  $x(\theta) = \phi(\theta), \forall \theta \in [-\tau, 0]$ ;
- (ii)  $x(t + \bar{\omega}) = x(t), t > 0$ , and  $x$  is a piecewise continuous function with discontinuity point of the first kind at time  $t + \bar{\omega}$ , and continuity on the left of every discontinuity point,  $k \in \{1, 2, \dots, p\}, r \in \{-1.0, 1, \dots\}$ ;

Let  $x(t, \phi)$  denote the solution for the system (1) at initial value  $\phi \in C^*$ .

*Definitions 3*

The  $\omega$ -periodic solution  $x(t, \phi^*)$  of the system (1) is called globally exponentially stable. If there is a constant  $\varepsilon > 0, k > 0$  for any initial function  $\phi \in C^*$ , the solution  $x(t, \phi)$  of the system (1) through the point  $(0, \phi)$  satisfies  $\|x(t, \phi) - x(t, \phi^*)\| \leq ke^{-\varepsilon t} \|\phi - \phi^*\|_\infty$ . among them

$$\|x(t, \phi) - x(t, \phi^*)\| = \sum_{i=1}^n |x_i(t, \phi) - x_i(t, \phi^*)|$$

$$\begin{aligned} \frac{d|x_i(t, \phi) - x_i(t, \phi^*)|}{dt} &= -a_i |x_i(t, \phi) - x_i(t, \phi^*)| \\ &+ f_i \left( \sum_{j=1}^n w_{ij}(t) x_j(t, \phi) + I_i \right) - f_i \left( \sum_{j=1}^n w_{ij}(t) x_j(t, \phi^*) + I_i \right) \text{sign}(x_i(t, \phi) - x_i(t, \phi^*)) \\ &+ \left( g_i \left( \sum_{j=1}^n m_{ij}(t) \int_{-\tau}^0 \eta_j(\theta) x_j(t + \theta, \phi) d\theta + I_i \right) \right. \\ &\left. - g_i \left( \sum_{j=1}^n m_{ij}(t) \int_{-\tau}^0 \eta_j(\theta) x_j(t + \theta, \phi^*) d\theta + I_i \right) \right) \text{sign}(x_i(t, \phi) - x_i(t, \phi^*)) \\ &\leq -a_i |x_i(t, \phi) - x_i(t, \phi^*)| + \alpha_i \sum_{j=1}^n w_{ij} |x_j(t, \phi) - x_j(t, \phi^*)| \\ &+ \beta_i \sum_{j=1}^n m_{ij}(t) \int_{-\tau}^0 \eta_j(\theta) |x_j(t + \theta, \phi) - x_j(t + \theta, \phi^*)| d\theta \end{aligned} \tag{3}$$

Therefore,

### 3. Main Work

*Lemma* Conditions (H1)-(H3) hold, and  $C$  is  $M$ -matrix, The system has a unique global exponentially stable  $\omega$ -periodic solution. Among them  $C = A - \alpha W - \beta MK$ .

Proof: Assume that  $x(t, \phi)$  and  $x(t, \varphi)$  are arbitrary two solutions for system (1). Because  $C = A - \alpha W - \beta MK$  is a  $M$ -matrix,  $C^T$  (transpose of the matrix  $C$ ) is also a  $M$ -matrix. According to the equivalent conditions given in section 2.1, there is a set of constants

$\xi_i > 0, j = 1, 2, \dots, n$ , such that

$$\xi_i(-a_i) + \sum_{j=1}^n \xi_j w_{ji} \alpha_j + \eta_i \sum_{j=1}^n \xi_j m_{ji} \beta_j < 0$$

Therefore, it is possible to select a constant  $\varepsilon > 0$ ;

$$\xi_i(\varepsilon - a_i) + \sum_{j=1}^n \xi_j \alpha_j w_{ji} + \sum_{j=1}^n \xi_j \beta_j m_{ji} \eta_i e^{\varepsilon t} < 0 \tag{2}$$

Constructing Lyapunov functional

$$\begin{aligned} V(t) &= \sum_{i=1}^n \xi_i |x_i(t, \phi) - x_i(t, \varphi)| \\ &+ \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) \int_{t+\theta}^t |x_j(v, \phi) - x_j(v, \varphi)| e^{\varepsilon(v-\theta)} dv d\theta \end{aligned}$$

When

$t \neq t_k + r\omega, k \in \{1, 2, \dots, p\}, r \in \{0, 1, 2, \dots\}, i \in \{1, 2, \dots, n\}$ ,

We could obtain the following from the (H1)

$$\begin{aligned}
 D^+V &\leq \sum_{i=1}^n \xi_i e^{\varepsilon t} \left( -a_i |x_i(t, \varphi) - x_i(t, \phi)| + \alpha_i \sum_{j=1}^n w_{ij} |x_j(t, \varphi) - x_j(t, \phi)| \right. \\
 &\quad \left. + \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) |x_j(t+\theta, \varphi) - x_j(t+\theta, \phi)| d\theta \right) + \varepsilon \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} |x_j(t, \phi) - x_j(t, \varphi)| e^{\varepsilon t} \int_{-\tau}^0 \eta_j(s) e^{-\varepsilon \theta} d\theta - \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} e^{\varepsilon t} \int_{-\tau}^0 \eta_j(s) |x_j(t+\theta, \phi) - x_j(t+\theta, \varphi)| d\theta \\
 &\leq e^{\varepsilon t} \sum_{i=1}^n \xi_i (\varepsilon - a_i) |x_i(t, \phi) - x_i(t, \varphi)| + e^{\varepsilon t} \sum_{j=1}^n \xi_j \alpha_j \sum_{i=1}^n w_{ij} |x_i(t, \phi) - x_i(t, \varphi)| + e^{\varepsilon t} \sum_{j=1}^n \xi_j \beta_j \sum_{i=1}^n m_{ji} |x_i(t, \phi) - x_i(t, \varphi)| \int_{-\tau}^0 \eta_j(\theta) e^{-\varepsilon \theta} d\theta \\
 &= e^{\varepsilon t} \sum_{i=1}^n \left\{ \xi_i (\varepsilon - a_i) + \sum_{j=1}^n \xi_j \alpha_j w_{ji} + \sum_{j=1}^n \xi_j \beta_j m_{ji} \eta_i e^{\varepsilon t} \right\} |x_i(t, \phi) - x_i(t, \varphi)|
 \end{aligned}$$

we could obtain the following from the (2)

$$D^+V \leq 0 \tag{4}$$

When

$$t \neq t_k + r\omega, \quad k \in \{1, 2, \dots, p\}, \quad r \in \{0, 1, 2, \dots\}, \quad i \in \{1, 2, \dots, n\},$$

$$\begin{aligned}
 V(t+0) &= \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t+0, \phi) - x_i(t+0, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) \int_{t+\theta}^t |x_j(v, \phi) - x_j(v, \varphi)| e^{\varepsilon(v-\theta)} dv d\theta
 \end{aligned}$$

$$\begin{aligned}
 V(t) &= \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) \int_{t+\theta}^t |x_j(v, \phi) - x_j(v, \varphi)| e^{\varepsilon(v-\theta)} dv d\theta
 \end{aligned}$$

Combined with conditions (H2)

$$\begin{aligned}
 V(t+0) - V(t) &= e^{\varepsilon t} \sum_{i=1}^n \xi_i (|x_i(t+0, \phi) - x_i(t+0, \varphi)| - |x_i(t, \phi) - x_i(t, \varphi)|) \\
 &= e^{\varepsilon t} \sum_{i=1}^n \xi_i (|I_{ik} + 1| - 1) |x_i(t, \phi) - x_i(t, \varphi)| \leq 0
 \end{aligned}$$

ie

$$V(t+0) \leq V(t) \tag{5}$$

Combined with conditions (4)

$$V(t) \leq V(0), \quad t \geq 0 \tag{6}$$

And

$$\begin{aligned}
 V(0) &= \sum_{i=1}^n \xi_i |x_i(t, \phi) - x_i(t, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) \int_{\theta}^0 |x_j(v, \phi) - x_j(v, \varphi)| e^{\varepsilon(v-\theta)} dv d\theta \\
 &= \sum_{i=1}^n \xi_i |\phi_i(0) - \varphi_i(0)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) \int_{\theta}^0 |\phi_j(v) - \varphi_j(v)| e^{\varepsilon(v-\theta)} dv d\theta \\
 &= \sum_{i=1}^n \xi_i \left( 1 + \frac{1}{\varepsilon} \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) (e^{-\varepsilon \theta} - 1) d\theta \right) \|\phi - \varphi\|_{\infty}
 \end{aligned}$$

However

$$\begin{aligned}
 V(t) &= \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) \int_{t+\theta}^t |x_j(v, \phi) - x_j(v, \varphi)| e^{\varepsilon(v-\theta)} dv d\theta \\
 &\geq \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| \\
 &\geq \min_{1 \leq i \leq n} \{\xi_i\} e^{\varepsilon t} \sum_{i=1}^n |x_i(t, \phi) - x_i(t, \varphi)|
 \end{aligned}$$

Notes

$$N(\varepsilon) = \left( \min_{1 \leq i \leq n} \{\xi_i\} \right)^{-1} \sum_{i=1}^n \xi_i \left( 1 + \frac{1}{\varepsilon} \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n m_{ij} \int_{-\tau}^0 \eta_j(\theta) (e^{-\varepsilon \theta} - 1) d\theta \right),$$

obviously,

$$N(\varepsilon) > 0,$$

So

$$\sum_{i=1}^n |x_i(t, \phi) - x_i(t, \varphi)| \leq N(\varepsilon) e^{-\varepsilon t} \|\phi - \varphi\|_{\infty} \tag{7}$$

So

$$\sum_{i=1}^n |x_i(t+\theta, \phi) - x_i(t+\theta, \varphi)| \leq N(\varepsilon) e^{-\varepsilon(t+\theta)} \|\phi - \varphi\|_{\infty}, \quad \forall \theta \in [-\tau, 0] \tag{8}$$

Notes

$$x_i(\phi)(\theta) = x(t + \theta, \phi), \quad \forall \theta \in [-\tau, 0]$$

Define the mapping  $P: C^* \rightarrow C^*, \forall \phi \in C^*$

$$P\phi = x_\omega(\phi)$$

Suppose that  $m$  is a positive integer not less than  $\frac{\ln(2N(\varepsilon))}{\varepsilon\omega}$ , in combination with (8)

$$\begin{aligned} \|P^m\phi - P^m\phi^*\|_\infty &= \|P^{m-1}x_\omega(\phi) - P^{m-1}x_\omega(\phi^*)\|_\infty \\ &= \|P^{m-2}x_\omega(x_\omega(\phi)) - P^{m-2}x_\omega(x_\omega(\phi^*))\|_\infty \\ &= \|P^{m-2}x_{2\omega}(\phi) - P^{m-2}x_{2\omega}(\phi^*)\|_\infty \\ &= \dots = \|x_{m\omega}(\phi) - x_{m\omega}(\phi^*)\|_\infty \\ &= \|x(m\omega + \theta, \phi) - x(m\omega + \theta, \phi^*)\|_\infty \\ &\leq N(\varepsilon)e^{-\varepsilon(m\omega + \theta)}\|\phi - \phi^*\|_\infty \\ &\leq \frac{1}{2}\|\phi - \phi^*\|_\infty \end{aligned}$$

Therefore,  $P^m: C^* \rightarrow C^*$  is the contractive mapping. By the principle of the contractive mapping, let  $\phi^*$  be  $P^m$  has a unique fixed point on  $C^*$ , so

$$P^m(P\phi^*) = P(P^m\phi^*) = P\phi^*$$

Therefore  $P\phi^*$  is also the fixed point of  $P^m: C^* \rightarrow C^*$ . By the uniqueness of the fixed point was  $P\phi^* = \phi^*$ , that is,

$$x_{t+\omega}(\phi^*) = x_t(\phi^*), \quad \forall t \geq 0$$

So

$$x(t + \omega, \phi^*) = x(t, \phi^*), \quad \forall t \geq 0$$

I.e.  $x(t, \phi^*)$  is  $\omega$ -periodic.

By the (7) known to the system (1) of any one solution  $x(t, \phi)$ , has

$$\sum_{i=1}^n |x_i(t, \phi) - x_i(t, \phi^*)| \leq N(\varepsilon)e^{-\varepsilon t} \|\phi - \phi^*\|_\infty \quad (9)$$

I.e. the periodic solution of the system (1) is globally exponentially stable.

## 4. Conclusion

This paper introduces the research status of artificial neural network, summarizes the research background and development of static recurrent neural network dynamic

system, and introduces the main work of this paper. Using the fixed point theory, M. The existence of periodic solutions and the global robust exponential stability of the static recursive neural network with variable delays and the existence of almost periodic solutions of the static recursive neural network of the partitioned time are studied by combining the properties of the matrix and the Lyapunov function combined with the inequality technique. Global exponential stability, the stability conditions of the corresponding problem are obtained respectively, and the results of the related research are generalized. Using Lyapunov. The stability of the quasi-static neural recursive neural network and the stability of the periodic solution are studied. The condition of the stationary static recursive neural network is obtained and the correctness of the condition is illustrated.

Considering the influence of stochastic perturbation on the dynamic behavior of static recurrent neural network, the static recursive neural network with time delay and the static recursive neural network with distributed time delay are studied by using the infinitesimal operator, Ito formula and the convergence theorem of martingales. Global critical exponential stability of quasi-static neural network with stochastic perturbation.

The static recursive neural network with Markovian modulation and the time-delay static recurrent neural network model considering both random perturbation and Markovian switching are studied. The linear matrix inequality, the finite state space Markov chain property and the Lyapunov-krasovskii function, the judgment condition of the global exponential stability of the system is obtained. Firstly, the global exponential stability problem of quasi-static neural network with time-delay and recursive neural network is studied by using the generalized Halanay inequality. Then the stability of the Markovian response sporadic static recurrent neural network is studied by combining the properties of Markov chain.

## 5. Insufficient and Prospect

- In the study of the stability of the static neural network equilibrium point in the paper, if the pulse is not considered, it is stable under the given judgment condition, that is, the time delay system without considering the pulse is stable, the system considering the pulse In the corresponding conditions are stable, but if you do not consider the pulse of the system is unstable, how to use the pulse control system makes the system stable, this is not resolved, I will try to solve this problem.
- The same problem, that is, does not consider the diffusion term is stable, under the corresponding conditions, consider the diffusion of static neural network system is still stable.
- Some of the examples in the article have not been matlab or other mathematical tools to simulate the results, which is what I need to improve the place.
- Mentioned in the pulse, random disturbance, Markov

chain, so you can also consider the pulse and random perturbation of these two factors, or taking into account the pulse and Markov chain these two factors, or these three factors at the same time, This time-delay static recurrent neural network is more general.

## Acknowledgments

I would like to thank Zhang Hong for guiding me in the process of writing the paper.

---

## References

- [1] Linshan Wang and Daoyi Xu. Global exponential stability of Hopfeild reaction-diffusion neural networks with time-varying delays. *Science in China (Series F)*, 2003, 46 (6): 466-474.
- [2] Linshan. Wang, Yiyi. Gao. Global exponential robust stability of reaction-diffusion interval neural networks with time-varying delays. *Phys. Lett. A*, 2006, 350: 342-348.
- [3] Linshan Wang and Daoyi Xu. Asymptotic behavior of a class of reaction-diffusion equations with delays. *Journal Mathematical Analysis and Applications*, 2003, 28 1 (2): 439-453 (SCI1DS: 691EF).
- [4] Linshan Wang, Yangfan Wang. Stochastic exponential stability of the delayed reaction-diffusion recurrent neural networks with Markovian jumping parameters. *Physics Letters A*, 2007, 356 (4), 346-352.
- [5] Linshan Wang, Yan Zhang, Zhe Zhang, Yangfan Wang. LMI-based approach for global exponential robust stability for reaction-diffusion uncertain neural networks with time-varying delay. *Chaos, Solitons and Fracals* 41 (2009) 900-905.
- [6] Linshan Wang, Ruojun Zhang. Global exponential stability of reaction-diffusion cellular neural networks with S-type distributed time delays. *Nonlinear Analysis: Real World Applications*, 2009, 10 (2): 1101-1113.
- [7] P. Balasubramaniam, R. Rakkiyappan. Global asymptotic stability of stochastic recurrent neural networks with multiple discrete delays and unbounded distributed delays. *Applied Mathematics and Computation*, 2008, 204 (2): 680-686.
- [8] Yah Lv, Wei Lv, Jianhua Sun. Convergence dynamics of stochastic reaction-diffusion recurrent neural networks with continuously distributed delays. *Nonlinear Analysis: Real World Applications*, 2008, 9 (4): 1590-1606.
- [9] Zidong Wang, Huisheng Shu, Jian'an Fang, Xiaohui Liu. Robust stability for stochastic Hopfield neural networks with time delays. *Nonlinear Analysis: Real World Applications*, 2006, 7 (5): 1119-1 128.
- [10] Yonghui Sun, Jinde Cao. pth moment exponential stability of stochastic recurrent neural networks with time-varying delays. *Nonlinear Analysis: Real World Applications*, 2007, 8 (4): 1171-1185.